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## **Analytical Traction Force Model Development for Soil-Tire Interaction: Incorporating Dynamic Contact Area and Parameter Analysis Using Taguchi Method**

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### **ABSTRACT**

Three parameters of soil, vertical load and tire play a role in determining vehicle traction force in the process of soil and tire interaction. The complexity of the situation and the variability of variables such as the soil-tire contact area and contact pressure make it difficult to develop traction force estimation models. In this study, the first step involved developing a traction force prediction model under the assumption of a variable contact area and contact pressure, and developing a mathematical model to predict off-road vehicle traction force. The obtained model includes seven parameters related to tire, load, soil and tire dynamics of wheel movement, which are vertical load, soil-tire contact length, tire width, slip, soil cohesion, soil shear deformation parameter and angle of internal shear resistance. A statistical population with five levels for each of its component parameters was created to study the impact of those parameters. The Taguchi method was used to examine the relationship between parameters and traction force. The Taguchi method is employed to determine the key factors that significantly impact a process, also it employs a systematic experimental design, to minimize the number of experiments needed. The results confirmed that all seven parameters had a significant impact on the amount of traction force and established the relative importance of their effects on one another. As a result, the tire width and slip parameters played the most and the least roles in improving traction force, respectively.

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## INTRODUCTION

Interaction models between tire and soil are used to analyze stresses, traction force, rolling resistance and energy. For off-road vehicle performance evaluation and soil compaction, the relationship between soil and tire is very important (Li, 2013; Schjønning et al., 2015). For many years, soil that has been compacted by vehicle traffic may stay that way (Schjønning et al., 2015). Many studies have been conducted in these fields and various experimental, semi-experimental and analytical models have been proposed. Analytical modeling of tire behavior with soil is relatively challenging due to the complicated behavior of the soil. To determine the fields of slip lines and associated stresses, Karafiath (1971) proposed an equilibrium differential equation based on the Moore-Columbus refractive index. The limitation of this method was that the solutions obtained were valid only for static mode. Upadhyaya and Wulfsohn (1993) showed that experimental traction force models are not able to provide an accurate understanding of the basic mechanics and they should be used with caution when assessing new tire position. The purpose of using analytical models is to predict vehicle behavior in the face of the soil without any need for experimental measurements. However, analytical models also require some soil parameters that can only be obtained through experimental measurements, such as cohesion and angle of shearing resistance of the soil. Bekker (1969) developed his model using cohesion measurement and angle of shearing resistance of the soil. The central concept of the Bekker's theory is that the vertical stress under a tire at a certain depth from the soil surface is equal to the stress obtained in the plate sinkage test at the same depth. Considering Bekker's arguments, Muro (2004) examined several relationships between tire and soil in which the traction force produced by the tire on the soil was reliant on the soil's maximum shear strength. One of the determining elements in the interaction between the soil and tire is the contact area. Based on tire geometry, Wulfsohn and

Upadhyaya (1992) developed a mathematical method for calculating the contact area on a rigid surface. The three-dimensional deformation of the soil was obtained using this method by fitting the soil deformation model coefficients at the joint area of the tire and the soil. Several points along the contact area between the tire and the terrain were experimentally extracted. Stress applied at each point determines how much the soil will deform there. The majority of earlier studies assumed that the stress distribution beneath the tire was uniform, which resulted in errors when calculating the values of stress and traction force. Additionally, they are not accurate enough estimate the degree of soil compression. A precise and correct analysis of the tire load distribution on the terrain is required to determine the actual amount of stress under the tire. The involvement of tires and soil cannot be fully and realistically understood by using uniform stress distributions or simplified contact areas (Schjønning et al., 2015). Keller et al. (2014) emphasized the use of non-uniform stress distributions in the contact area between tire and soil. Various models have been proposed to estimate the contact area between soil and tire. Based on the constituent laws that describe the mechanical behavior of soil and tires, analytical models have the potential to examine the mechanics of soil-tire interaction in detail. Obtaining the contact area between the tire and the soil is the first step in determining the parameters of stress, traction force, rolling resistance, and energy (performance parameters) of the tire with the soil (Taghavifar and Mardani, 2017). The simplest methods assume that the traction force device's contact area is a rectangle, which is a reasonable approximation for tracked vehicles but inaccurate for tires (He, 2020). Some studies, like those conducted by Youssef and Ali (1982), argue that the tire's contact area is elliptical. Contact areas seldom form an ellipse unless the ground is quite elastic and no deformation remains after the tire has passed over the surface (Hallonborg, 1996). Upadhyaya and Wulfsohn (1990) formulated a two-dimensional mathematical equation in an elliptical shape to

describe the contact area of a pneumatic tire on a rigid surface, taking into account the tire's geometry and characteristics. When the surface is rigid and the tire pressure is high, it is possible to approximate the contact area with a circular shape. Considering a softer surface and lower tire pressure, the geometry of the contact area can be regarded as more dependable. A model was proposed by Keller (2005) to predict both the contact area and the vertical stress distribution underneath the tire. The tire parameters in this model are interconnected with the properties of the contact area and stress distribution between the tire and the soil. The contact area is characterized as a superellipse shape, drawing inspiration from the Hallonborg model. Subsequent studies utilized the superellipse model to depict the contact area with minimal alterations (Lamandé and Schjønning, 2008; Schjønning et al., 2015, 2008). The super ellipse model can accurately estimate the contact area between tire and soil that is defined by Keller (2005) as follows:

$$\left| \frac{x}{a} \right|^n + \left| \frac{y}{b} \right|^n = 1 \quad (1)$$

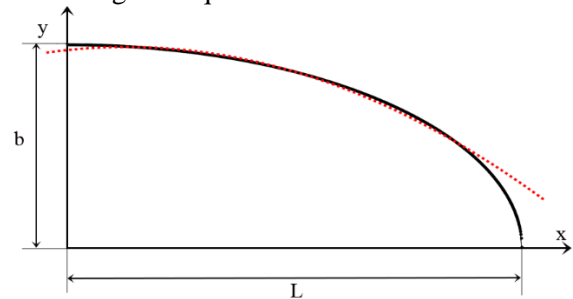
Where  $a$ , and  $b$  are the half of the major and minor diameters in the ellipse and  $n$  is the exponent. For larger exponent, the contact area gets closer to the rectangle. The disadvantage of this method is that it is difficult to obtain exact values of  $n$  to describe the contact area.

In this study, an attempt was made to calculate the tires' traction force by taking into account more precise circumstances. The applied approach includes the application of variable pressure on the contact area of the wheel and the soil, as well as a more realistic approximation of the contact surface. An evaluation of the independent and interaction effects of these parameters on tire traction force, rolling resistance, and energy consumption has been conducted based on of the obtained model and a statistical population has been created within the typical range of parameters making up the model.

## MATERIALS AND METHODS

### Calculating the contact area between the tire and soil

In this study, a mathematical method is presented to obtain the contact area. This method divides the tire's contact area with the soil into four symmetrical parts, each measuring two by two. This division is such that the contact width is the same in both halves, but the contact length may not be equal. A quadratic relation fits a separate ellipse to each half of the common boundary between the soil that is undisturbed and the soil that is underneath the tire. The front and back surface borders of the quadratic equation can be integrated to determine the contact area accurately. The contact area will be symmetrical and take the shape of an ellipse in the particular case where the contact area is a rigid surface. It is sufficient to perform the calculations for a quarter of the surface in this instance because both the front and back halves will be symmetrical. The intersection of the transverse and longitudinal axes in the middle of the contact area is taken into account as the center of coordinates. It is assumed to be symmetrical with concerning to the longitudinal axis of the tire when calculating the contact area. This contact area is shown in Figure 1 Where the dotted line diagram shows the fitted parabola at the end of the contact area,  $b$  is the contact width and the contact length is equal to  $L$ .



**Figure 1.** The curve fitted to a quarter of the soil-tire contact area

The fitted curve is a quadratic equation with constant coefficients.

$$y = a_0x^2 + a_1x + a_2 \quad (2)$$

To obtain the fitted curve coefficients, it is sufficient to replace the boundary points in the curve equation. By solving the obtained equations, the fitted curve coefficients are obtained as follows:

$$a_0 = \frac{(2 - 2\sqrt{3})b}{L^2} \quad (3)$$

$$a_1 = \frac{(2\sqrt{3} - 3)b}{L} \quad (4)$$

$$a_2 = b \quad (5)$$

By replacing the constants of Eq. (3), (4), and (5) with Eq. (2), the fitted curve equation of the contact area of the tire and soil is defined as follows:

$$y = \frac{(2 - 2\sqrt{3})b}{L^2}x^2 + \frac{(2\sqrt{3} - 3)b}{L}x + b \quad (6)$$

Of course, the calculations have been conducted for the front half of the contact area and for the rear half, the calculations must be repeated according to the length of the contact and they must be finally added together.

The contact area of the tire can be written as follows:

$$A = \int_0^L \left( \frac{(2 - 2\sqrt{3})b}{L^2}x^2 + \frac{(2\sqrt{3} - 3)b}{L}x + b \right) dx \quad (7)$$

$$A = \frac{(2\sqrt{3} + 1)Lb}{6} = 0.744Lb \quad (8)$$

### Calculating the normal stress on the contact area

In most previous studies, the stress distribution at the contact area has been considered uniform, which is not an exact approximation of the actual stress. Bekker (1969) introduced a semi-experimental pressure-sinkage relationship for homogeneous soils. In this regard, Becker measured soil parameters using soil indicators. Most methods introduced to predict stress under the tire consider the soil as an elastic medium. The relation proposed by Bekker for pressure-sinkage in terra mechanics by applying force on rigid rectangular plates with different dimensions

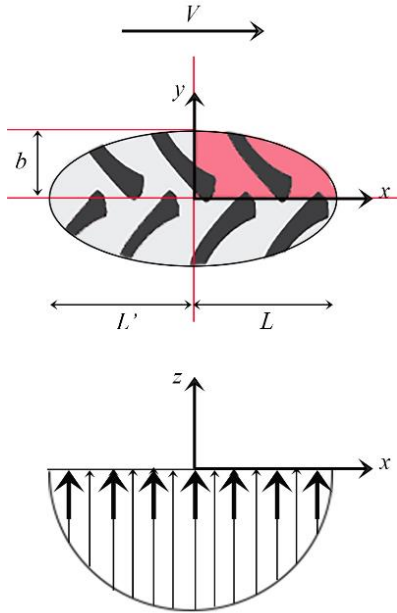
and measuring their penetration into the soil is defined as follows:

$$P = \left( \frac{K_c}{b} + K_\phi \right) . Z^n \quad (9)$$

In this equation,  $P$  is pressure;  $b$  is the radius of a circular plate or the smaller dimension of a rectangular plate;  $n$ ,  $K_c$  and  $K_\phi$  are pressure-sinkage parameters for the Bekker equation. The main feature of this relation was the separation of the soil shear deformation parameter into two parts  $K_c$  and  $K_\phi$ . The equation proposed by Bekker has been validated in many soils with different plates and traction force devices. It is currently one of the most widely used methods in this field. Wong (2008) reported the values of  $K_c$  and  $K_\phi$  for some soils with specific moisture content in a table for use under similar conditions. However, the measurement of soil parameters seems necessary for studies due to highly variable soil conditions. Kacigin and Guskov (1968) showed that hyperbolic functions could be substituted for exponential laws used for deformation resulting from soil stress. The obtained hyperbolic equation had two soil-dependent parameters. This relationship was proposed based on the maximum soil shear stress and soil deformation coefficient. These parameters in Kacigin and Guskov's (1968) study are solely dependent on the type of soil and the amount of moisture, regardless of the size and shape of the plates used in Bekker's model. The LSA analytical model was created by Lyasko (2010). The LSA model was developed to predict pressure-sinkage curves for specific soil conditions for steady-state penetration, in which fixed soil parameters are measured experimentally before calculations by conventional soil mechanics methods.

There are few analytical methods can be used to model the variable stress in the tire-soil contact area. Most of the methods introduced experimentally or semi-experimentally estimate the amount of stress. The real traction force under the tire varies as a parabola in the longitudinal direction, which is zero at the beginning and end of the tire's involvement with the tire. The assumption of constant stress on the soil seems

away from reality and can cause significant calculation errors. On the other hand, most of the existing stress-displacement relations model the interaction of sprocket vehicles with the soil.



**Figure 2.** Variable pressure distribution during tire contact with soil

In this study, it was tried to provide a mathematical relation to define the stress between pneumatic tires and soil. Assuming that the amount of stress distribution is a quadratic equation of the contact length, the amount of force applied to the soil can be calculated by the following equation:

$$F = \int_0^L \int_0^y Mx^2 dA = 0.1565ML^3b \quad (10)$$

In this relation,  $M$  is a constant coefficient that shows the stress distribution. If the vertical load on the tire axle is  $W$ , then the amount of force applied to a quarter of the contact area will be equal to  $0.25W$ . Based on this assumption, the stress coefficient can be obtained:

$$F = 0.25W \quad (11)$$

Where  $F$  is the load on the contact area, from the equation of relations (10) and (11), the value of  $M$  is obtained as:

$$M = \frac{1.67W}{bL^3} \quad (12)$$

Where  $L$  is the contact length in the first quarter, and  $b$  is half the width of the contact.

By obtaining the stress coefficient, the stress equation can be written as follows:

$$\sigma = \frac{1.67W}{bL^3} x^2 \quad (13)$$

### Calculating the traction force

Shmulevich and Osetinsky (2003) proposed a semi-experimental tire traction force model. The central part of the traction force model was the modeling of tire and soil contact lines as a Parabolic. The Janusi and Hanomoto's stress-displacement model and the Bekker's pressure-sinkage model were used in the Shmulevich and Osetinsky's traction force model. Schjønning et al. (2015) proposed a model for distributing the actual tire pressure relative to the tire pressure recommended by the manufacturer. The effect of slipping at the point of maximum pressure on the tire-soil contact area was not taken into account in this model. Instead, the slip effect is taken into account by the radial stress distribution model, which was initially proposed for tire-soil interaction. This model was used in accordance with the off-road's traction force model developed by Senatore and Sandu (2011).

For the pressure-sinkage and shear stress-displacement models that are the foundation for the soil-tire traction force models, the traction force model includes parameterization of pressure-sinkage and shear stress-displacement, which are tested using pressure plates and shear tests, respectively (He, 2020).

In this study, Bekker's pressure-sinkage model and Janusi and Hanomoto's shear stress-displacements are used as the fundamental traction force models for calculations. This model is defined by the following relation:

$$\tau = \tau_{\max} \left[ 1 - \exp\left(-\frac{j}{k}\right) \right] \quad (14)$$

Where  $j$  and  $K$  are shear displacements and shear deformation parameters of soil, respectively. Replacing the maximum shear stress value ( $\tau_{\max}$ ), the following relations can be written:

$$\tau_{\max} = c + \frac{1.67W}{bL} \tan(\varphi) \quad (15)$$

$$\tau = \left[ c + \frac{1.67W}{bL} \tan(\varphi) \right] \left[ 1 - \exp\left(-\frac{j}{k}\right) \right] \quad (16)$$

By integrating the Eq. (16) on the area, it is possible to obtain the maximum traction force that can be applied to the soil:

$$H = \iint \tau dA \quad (17)$$

$$\begin{aligned} H = & W \tan(\varphi) \left[ 0.261 - 3.34\left(\frac{k}{iL}\right)^3 - 4.65\left(\frac{k}{iL}\right)^4 + 58.8\left(\frac{k}{iL}\right)^5 \right] \\ & + W \tan(\varphi) \left[ \frac{-0.005k}{iL} - 4.135\left(\frac{k}{iL}\right)^2 - 21.45\left(\frac{k}{iL}\right)^3 - 54.15\left(\frac{k}{iL}\right)^4 - 58.8\left(\frac{k}{iL}\right)^5 \right] \exp\left(\frac{-iL}{k}\right) \\ & + bc \left[ 0.744L - \frac{k}{i} - \frac{0.464k^2}{i^2L} + \frac{2.92k^3}{i^3L^2} + \left[ \frac{0.004k}{i} - \frac{2.456k^2}{i^2L} - \frac{2.92k^3}{i^3L^2} \right] \exp\left(\frac{-iL}{k}\right) \right] \end{aligned} \quad (19)$$

The parameters of the soil, tire and slip are functions of the obtained traction force relation. The seven constituent parameters of this relation, which has a mathematical form, have been discussed in this study.

## RESULTS AND DISCUSSION

### Data Analysis using the Taguchi Method

A statistical population covering the typical range for each parameter has been created based on the mathematical model obtained for traction force to investigate the impact of parameters on traction force. About vehicle and soil characteristics in relation, factors affecting traction force were replaced in the current study at five levels, and their placement and impact analysis have been investigated using the Taguchi analysis method taking into account the larger response value.

$$H = \int_0^x \int_0^y \left[ c + \frac{1.67W}{bL} \tan(\varphi) \right] \left[ 1 - \exp\left(-\frac{j}{k}\right) \right] dy dx \quad (18)$$

The equation obtained for traction force includes seven parameters related to soil, tire and dynamics of wheel movement, which are vertical load, tire width, area length, soil cohesion, friction angle, slip and shear deformation parameter of soil.

For every combination of factor levels, the signal-to-noise (S/N) ratio is computed. The formula for calculating the S/N ratio, where larger values are considered better, involves using base 10 logarithms (Eq. 20):

$$SN = -10 \log\left(\frac{1}{N} \sum_{i=1}^n \frac{1}{y^2}\right) \quad (20)$$

In this context, "y" represents the response for a specific combination of factor levels, while "n" refers to the number of responses within that factor level combination.

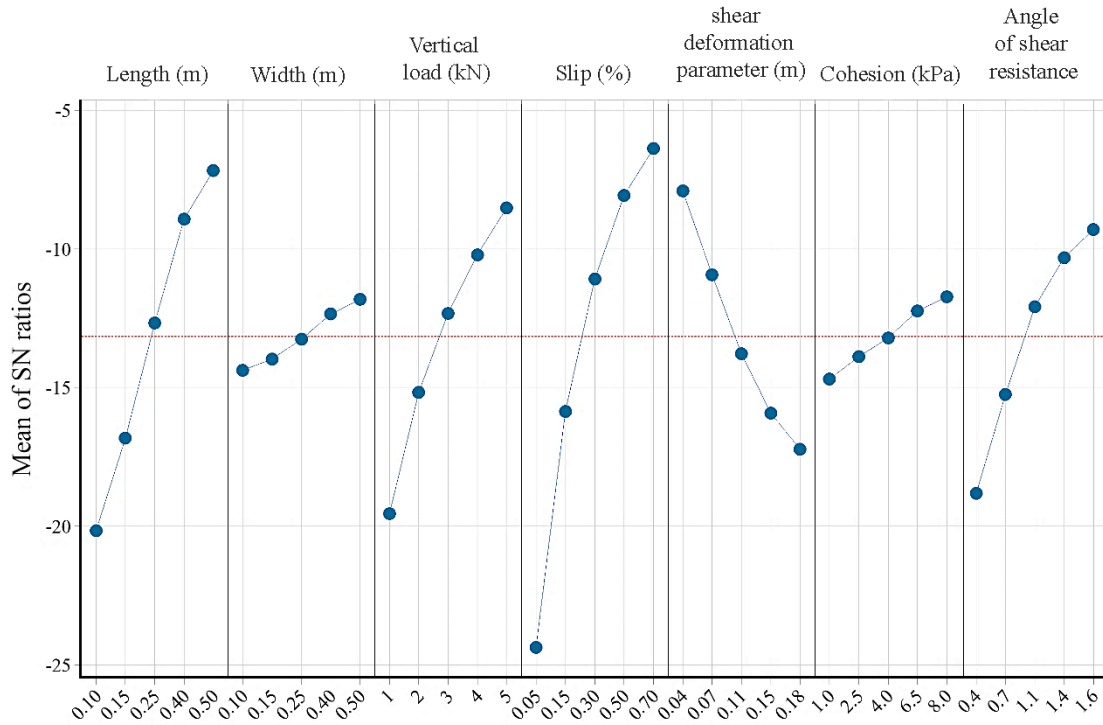
In defining the levels of the parameters, it has been tried to consider the usual and logical range of each parameter. The solution obtained from the parameters in Eq. (19) for traction force has been extracted in Minitab software and the obtained results are presented. Table 1 shows the desired factors and their levels.

**Table 1.** Influencing parameters and their levels.

Level	Length(m)	Width(m)	Weight (KN)	Slip	Shear deformation parameter (m)	Cohesion (kPa)	Angle of shear resistance (deg)
1	0.05	0.05	1	0.05	0.04	1	21.8
2	0.15	0.15	2	0.15	0.07	2.5	35
3	0.25	0.25	3	0.3	0.11	4	47.7
4	0.4	0.4	4	0.5	0.15	6.5	54.5
5	0.5	0.5	5	0.7	0.18	8	59.5

The *S.N.* diagram can be seen in Figure 3 From the diagrams obtained for the main effect, it can be seen the amount of traction force increases with an increase in parameters of slip, contact length, weight on the tire, shear strength, cohesion and contact width, respectively. Also, traction force decreases with increasing the shear

deformation parameter of the soil. Similar information about these parameters' impact on traction force can be found in sources like Taheri et al. (2015). The analysis also compares how each of the seven parameters affects traction force.



Signal-to-noise: Larger is better

**Figure 3.** Results of Taguchi analysis for the effect of seven parameters on traction force.

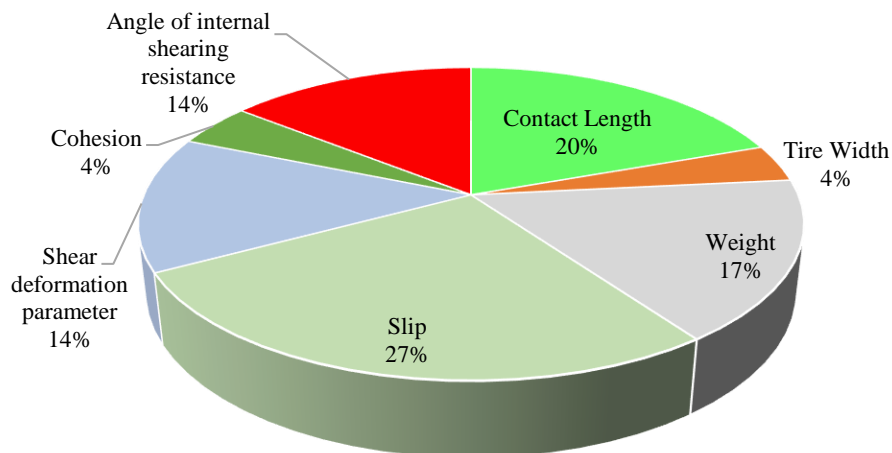
Table 2 shows the delta and the ranking of the parameters based on how effective they are in relation to traction force, as well as the average values of the signal-to-noise ratio for each level. According to this table, the highest effect on traction force with higher delta is related to slip and its lowest value for contact width. ten Damme et al. (2020), found similar results in their study of the effects of tire width on stress. (Delta value is calculated as the difference between the maximum and minimum value of a parameter.). Table 2 thus, shows that slip is the most important parameter for traction force. This conclusion can also be drawn from Figure 3, which shows the slip's obvious influence on traction force. It is **Table 2.** Response Table for Signal-to-Noise Ratios

also evident from the shape's variation between the minimum and maximum diagrams. Following slip, traction force is significantly influenced by contact length, tire vertical load, and soil shear strength, respectively. Additionally, the amount of traction force is significantly influenced by the soil's shear deformation parameter. As can be seen, this parameter has an inverse effect. It is also observed that the parameters of contact width and soil cohesion have less effect on traction force, which is consistent with the delta values obtained in Table 2.

Level	Length	Width	Weight	Slip	Shear deformation parameter	Cohesion	Angle of shear resistance
1	-20.155	-14.373	-19.546	-24.372	-7.912	-14.694	-18.812
2	-16.824	-13.968	-15.165	-15.863	-10.930	-13.889	-15.244
3	-12.674	-13.254	-12.325	-11.082	-13.781	-13.206	-12.083
4	-8.928	-12.345	-10.207	-8.064	-15.913	-12.238	-10.310
5	-7.175	-11.815	-8.513	-6.375	-17.220	-11.729	-9.306
Delta	12.981	2.559	11.033	17.997	9.308	2.965	9.506
Rank	2	7	3	1	5	6	4

In Figure 4 the effect of parameters on traction force is shown in a pie chart in percentage to gain

a better understanding of the effect of parameters on traction force.



**Figure 4.** Chart of the contribution of traction force equation parameters in tire traction force.

## CONCLUSION

The purpose of this study was to investigate the impact of effective parameters on traction force. First, an attempt was made to create a more precise model of the pneumatic tire traction force equation that included all of its influencing factors. The impact of each factor, including tires, loads, soil, and dynamics of wheel movement, is taken into account in this model. In contrast to most studies, the pressure distribution at the soil-tire contact area is taken into account as a variable when estimating the traction force equation. Additionally, it is assumed that the value of the contact area is non-constant and dependent on the environment. The effect of constituent parameters on traction force in the obtained model was investigated using the Taguchi

method. According to the Taguchi analysis, the seven traction force model parameters that have the greatest impact on traction force are slip, contact length, vertical load, angle of internal shear strength, soil shear deformation parameter, cohesion, and tire width.

## REFERENCES

- Bekker, M. G. (1969).** Off-Road Locomotion. *Ordnance*, 53(292), 416-418. <https://doi.org/10.2307/3612491>
- Hallonborg, U. (1996).** Super ellipse as tyre-ground contact area. *Journal of Terramechanics*, 33(3), 125-132. [https://doi.org/10.1016/S0022-4898\(96\)00013-4](https://doi.org/10.1016/S0022-4898(96)00013-4)
- He, R. (2020).** *Systematic Tire Testing and Model Parameterization for Tire Traction on Soft Soil* (Doctoral dissertation, Virginia Tech).



- Kacigin, V. V., & Guskov, V. V. (1968).** The basis of tractor performance theory: Part 1—General laws of soil strength and deformation. *Journal of terramechanics*, 5(3), 43-66. [https://doi.org/10.1016/0022-4898\(68\)90080-3](https://doi.org/10.1016/0022-4898(68)90080-3)
- Karafiath, L. L. (1971).** Plasticity theory and the stress distribution beneath wheels. *Journal of terramechanics*, 8(2), 49-60. [https://doi.org/10.1016/0022-4898\(71\)90025-5](https://doi.org/10.1016/0022-4898(71)90025-5)
- Keller, T. (2005).** A model for the prediction of the contact area and the distribution of vertical stress below agricultural tyres from readily available tyre parameters. *Biosystems engineering*, 92(1), 85-96. <https://doi.org/10.1016/j.biosystemseng.2005.05.012>
- Keller, T., Berli, M., Ruiz, S., Lamandé, M., Arvidsson, J., Schjøning, P., & Selvadurai, A. P. (2014).** Transmission of vertical soil stress under agricultural tyres: Comparing measurements with simulations. *Soil and Tillage Research*, 140, 106-117. <https://doi.org/10.1016/j.still.2014.03.001>.
- Lamandé, M., & Schjøning, P. (2008).** The ability of agricultural tyres to distribute the wheel load at the soil–tyre interface. *Journal of Terramechanics*, 45(4), 109-120. <https://doi.org/10.1016/j.jterra.2008.09.004>
- Li, H. (2013).** *Analysis of off-road tire-soil interaction through analytical and finite element methods* (Doctoral dissertation, Kaiserslautern, Technische Universität Kaiserslautern, Diss., 2013).
- Lyasko, M. (2010).** LSA model for sinkage predictions. *Journal of Terramechanics*, 47(1), 1-19. <https://doi.org/10.1016/j.jterra.2009.06.004> .
- Muro, T., 2004.** *Terramechanics: land locomotion mechanics.* CRC Press. <https://doi.org/10.1201/9780203024157>
- Schjøning, P., Lamandé, M., Tøgersen, F. A., Arvidsson, J., & Keller, T. (2008).** Modelling effects of tyre inflation pressure on the stress distribution near the soil–tyre interface. *Biosystems Engineering*, 99(1), 119-133. <https://doi.org/10.1016/j.biosystemseng.2007.08.005>
- Schjøning, P., Stettler, M., Keller, T., Lassen, P., & Lamandé, M. (2015).** Predicted tyre–soil interface area and vertical stress distribution based on loading characteristics. *Soil and Tillage Research*, 152, 52-66. <https://doi.org/10.1016/j.still.2015.03.002>
- Senatore, C., & Sandu, C. (2011).** Off-road tire modeling and the multi-pass effect for vehicle dynamics simulation. *Journal of Terramechanics*, 48(4), 265-276. <https://doi.org/10.1016/j.jterra.2011.06.006>
- Shmulevich, I., & Osetinsky, A. (2003).** Traction performance of a pushed/pulled drive wheel. *Journal of Terramechanics*, 40(1), 33-50. <https://doi.org/10.1016/j.jterra.2003.09.001>.
- Taghavifar, H., & Mardani, A. (2017).** Off-road vehicle dynamics. *Studies in Systems, Decision and Control*, 70, 37. [https://doi.org/10.1007/978-3-319-42520-7\\_3](https://doi.org/10.1007/978-3-319-42520-7_3).
- Taheri, S., Sandu, C., Taheri, S., Pinto, E., & Gorsich, D. (2015).** A technical survey on Terramechanics models for tire–terrain interaction used in modeling and simulation of wheeled vehicles. *Journal of Terramechanics*, 57, 1-22. <https://doi.org/10.1016/j.jterra.2014.08.003>.
- ten Damme, L., Stettler, M., Pinet, F., Vervae, P., Keller, T., Munkholm, L. J., & Lamandé, M. (2020).** Construction of modern wide, low-inflation pressure tyres per se does not affect soil stress. *Soil and Tillage Research*, 204, 104708. <https://doi.org/10.1016/j.still.2020.104708>.
- Upadhyaya, S. K., & Wulfsohn, D. (1993).** Traction prediction using soil parameters obtained with an instrumented analog device. *Journal of terramechanics*, 30(2), 85-100. [https://doi.org/10.1016/0022-4898\(93\)90022-P](https://doi.org/10.1016/0022-4898(93)90022-P)
- Upadhyaya, S. K., & Wulfsohn, D. (1990).** Relationship between tire deflection characteristics and 2-D tire contact area. *Transactions of the ASAE*, 33(1), 25-0030. <https://doi.org/10.13031/2013.31288>
- Wong, J. Y. (2022).** *Theory of ground vehicles.* John Wiley & Sons. <https://doi.org/10.1002/9781119719984>.
- Wulfsohn, D., & Upadhyaya, S. K. (1992).** Determination of dynamic three-dimensional soil-tyre contact profile. *Journal of terramechanics*, 29(4-5), 433-464. [https://doi.org/10.1016/0022-4898\(92\)90046-M](https://doi.org/10.1016/0022-4898(92)90046-M).
- Youssef, A.-F.A., & Ali, G.A. (1982).** Determination of soil parameters using plate test. *Terramechanics*, 19, 129-147. [https://doi.org/10.1016/0022-4898\(82\)90016-7](https://doi.org/10.1016/0022-4898(82)90016-7).